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15MAT41

Fourth Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. Use statistical table is permitted.

Module-1

1 a. Using Taylor's series method, solve $dy = (xy - 1)dx$, $y(1) = 2$ at $x = 1.02$ considering upto 3rd degree term. **(05 Marks)**

b. Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ at the point $x = 0.2$ by taking step length $h = 0.2$. **(05 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at $x = 0.8$ by Adams – Bashforth predictor – corrector method. **(06 Marks)**

OR

2 a. Using modified Euler's method, find an approximate value of y when $x = 0.1$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$. Take $h = 0.1$ and perform three iterations. **(05 Marks)**

b. Solve $\frac{dy}{dx} = 2y + 3e^x$ $y(0) = 0$ using Taylors series method an find $y(0.1)$. **(05 Marks)**

c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ the data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. **(06 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right)$ $y(0) = 1$, $y'(0.1) = 0.5$, evaluate $y(0.1)$ using 4th order – Runge – Kutta method. **(05 Marks)**

b. Express $f(x) = x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(05 Marks)**

c. If α and β are the roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Using the Milne's method obtain the approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0.1) = 0.1$. Given :

$y(0.1) = 1.03995$ $y'(0.1) = 0.6955$ $y(0.2) = 1.138036$
$y'(0.2) = 1.258$ $y(0.3) = 1.29865$ $y'(0.3) = 1.873$ **(05 Marks)**

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. State and prove Rodrigue's formula. **(06 Marks)**

Module-3

5 a. Derive Cauchy's Riemann equations in Cartesian form. **(05 Marks)**

b. Using Cauchy's residue theorem evaluate the integral $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$. **(05 Marks)**

c. Find the bilinear transformation which maps the points Z = 0, i, ∞ onto the points W = 1, –i, –1, respectively. Find the invariant points. **(06 Marks)**

OR

6 a. State and prove Cauchy's theorem. **(05 Marks)**

b. Given $u - v = (x - y)(x^2 + 4xy + y^2)$ find the analytic function $f(z) = u + iv$. **(05 Marks)**

c. Discuss the transformation $W = e^z$. **(06 Marks)**

Module-4

7 a. Derive mean and variance of the Binomial distribution. **(05 Marks)**

b. The probability that an individual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals more than 2 will get a bad reaction. **(05 Marks)**

c. The joint probability distribution of two random variable X and Y as follows :

x \ y	–2	–1	4	6
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine :
i) Marginal distribution of X and Y
ii) Covariance of X and Y
iii) Correlation of X and Y. **(06 Marks)**

OR

8 a. Derive mean and standard deviation of exponential distribution. **(05 Marks)**

b. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours. Out of 2500 bulbs find the number of bulbs that are likely to last between 1900 and 2100 hours. Given that $P(0 < z < 1.67) = 0.4525$. **(05 Marks)**

c. The joint probability distribution of two random variable X and Y as follows :

x \ y	–4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine :
i) Marginal distribution of X and Y
ii) Covariance of X and Y
iii) Correlation of X and Y. **(06 Marks)**

Module-5

9 a. Explain the following terms :

i) Null hypothesis

ii) Type I and Type II error

iii) Significance level. **(05 Marks)**

b. Find he student 't' for the following variables values in a sample of eight – 4, –2, –2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. **(05 Marks)**

c. Find the fixed probability vector of the regular stochastic matrix :

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}.$$ **(06 Marks)**

OR

10 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. **(05 Marks)**

b. A set of five similar coins is tossed 320 times and the result is

Number of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution for v = 5 we have $\chi^2_{0.05} = 11.07$. **(05 Marks)**

c. A students study habits are as follows. If he studies one night, he is 60% sure not study the next night. On the other hand if he does not study one night, he is 80% sure not to study the next night. In the long run how often does he study? **(06 Marks)**

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